HAEF IB – FURTHER MATH HL

TEST

CALCULUS

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Name:_________________________________

Questions

1. [Maximum mark: 10]

(a) The Cartesian equation of the unit circle is given by $x^2 + y^2 = 1$.

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}$$ [2]

(b) Consider now the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y} \quad \text{with } y = 1 \text{ when } x = 0$$

Show that the solution is the unit circle given above

(i) by using separable variables. [2]

(ii) by letting $u = \frac{y}{x}$ and solving it as a homogeneous differential equation. [6]

2. [Maximum mark: 12]

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y} \qquad \text{with } y = 1 \text{ when } x = 0$$

(a) Show that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{x^2 + y^2}{y^3}$ by using implicit differentiation. [3]

(b) **Hence** find the Maclaurin series of the solution up to and including the x^2 term. [2]

Use the series in (b) to find an approximation for y when $x = 0.5$ [1]

(c) For the same differential equation use Euler's method with step value $h = 0.1$ to obtain an approximation for y when $x = 0.5$. [4]

(d) The exact solution of the differential equation is the unit circle $x^2 + y^2 = 1$.

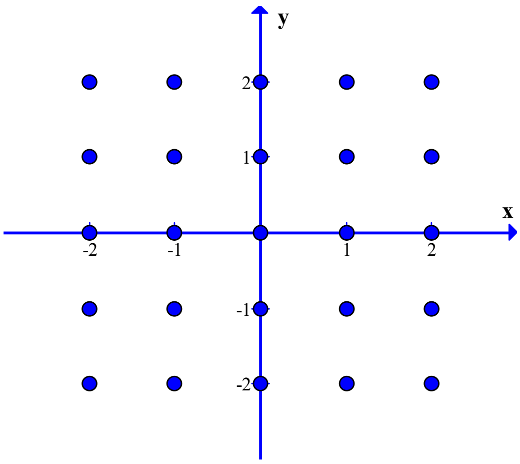
Find the errors of the approximation in (b) and (c) given that $y > 0$. [2]

3. [Maximum mark: 7]

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{y}$$

Copy the following diagram



(a) On your diagram sketch the isoclines $-\frac{x}{y} = c$ for $c = -1, 1$. [2]

(b) Sketch the slope field on the same diagram (ignore the origin $(0,0)$). [3]

(c) On the slope field sketch the solution that passes through the point $(0,1)$. [2]

4. [Maximum mark: 8]

Consider the infinite power series $\sum_{n=1}^{\infty} \frac{x^n}{7^n n^7}$

(a) Find the **radius** of convergence. [4]

(b) Find the **interval** of convergence. [3]

(c) **Hence** find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2017)^n}{7^n n^7}$. [1]

5. [Maximum mark: 5]

Find the derivatives of the following functions with respect to x

(a) $\int_{2017}^{x} \frac{1}{t^4+1}dt$. [1]

(b) $\int_{\sqrt{x}}^{x} \frac{1}{t^4+1}dt$. [4]

6. [Maximum mark: 8]

(a) Use the **mean value theorem** for $f(x)=\ln x$ in the interval $[1, x]$ to show that

$$(\ln x)+1 < x, \quad \text{for } x > 1$$ [4]

(b) By using a similar argument show that the same inequality holds for $0 < x < 1$. [3]

(c) Deduce that

$$(\ln x)+1 \le x, \quad \text{for any } x > 0$$ [1]