HAEF IB - MATH HL
TEST 6
STATISTICS AND PROBABILITY

Date: 19 October 2018
by Christos Nikolaidis

Paper 1: Without GDC

Name:_____

Marks: /40

Questions

1. [Maximum mark: 4]

(a) Given that $P(A' \cup B) = 0.53$ find $P(A \cap B')$. [2]

(b) Show that mutually exclusive, non-empty events are **not** independent. [2]

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2. [Maximum mark: 6]

Given that $P(A) = 0.7$, $P(B) = 0.2$ and $P(A' \cup B') = 0.9$, find

(a) $P(A \cup B)$ [3]

(b) $P(A' \mid A \cup B)$ [3]

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3. [Maximum mark: 6]

Let $P(A \mid B) = 0.6$ and $P(B \mid A) = 0.5$. Given that $P(A' \cap B') = 0.2$, find $P(B' \mid A')$

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4. [Maximum mark: 14]

The probability density function of a continuous random variable X is given by

$$f(x) = \frac{k}{x^2 + 4}, \quad \text{for } x \in [0,2]$$

(a) Find the value of k. [3]

(b) Show that the function f is decreasing in the domain $x \in [0,2]$. [2]

(c) Find the mode of X. [1]

(d) Find $E(X)$ in the form $\frac{\ln a}{\pi}$ [4]

(e) Find $E(X^2)$ [4]

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5. [Maximum mark: 10]

The probability distribution of a discrete random variable X is given by

$$P(X = x) = \frac{k}{x^2 + 2}, \quad \text{for } x = 0,1,2$$

(a) Find the value of k [3]

(b) Find $Var(X)$ [5]

(c) Find the mode of X [1]

(d) Find the median of X [1]

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Paper 2: With GDC

Name:_____

Marks: /40

Questions

6. [Maximum mark: 6]

Consider the following grouped data

x	**frequency**	**cumulative frequency**
[0,10[	15	15
[10,20[		30
[20,30[		40
[30,40[		70
[40,50]		80

Find

(a) The number of values of x in the interval [10,30[[2]

(b) The mean [1]

(c) The variance [2]

(d) The median [1]

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7. [Maximum mark: 8]

The probability density function of a continuous random variable X is given by

$$f(x) = \frac{k}{x^3 + 4}, \quad \text{for } x \in [0,2]$$

Find the following, correct to 3 s.f.

(a) the value of k [2]

(b) the values of $E(X)$ and $Var(X)$ [4]

(c) the median of X [2]

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8. [Maximum mark: 8]

A box contains 10 **red** balls and 10 **black** balls.

Vrasidas selects 3 balls (without replacement).

(a) Find the probability that only red balls are selected. [2]

(b) Find the probability that only one red ball selected. [2]

Melpomeni selects 4 balls (without replacement).

(c) Find the probability that all balls have the same color. [2]

(d) Find the probability that the red balls are more that the black ones. [2]

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9. [Maximum mark: 6]

Given that $P(B) = 0.2$, $P(A \mid B) = 0.7$ and $P(A \mid B') = 0.5$ find $P(B' \mid A')$.

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10. [Maximum mark: 6]

40 families are reviewed for the number of pets they own. Part of the results is shown below

Number of pets	**number of families**
0	14
1	
2	
3	
more than 3	none

Given that the mean is 1.1 and the variance is 1.04 complete the table above.

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11. [Maximum mark: 6]

A box contains 1 red ball, 1 green ball and 8 white balls. Players pay a fee of 2€ to participate in the following game. They select two balls

1st prize: RED-GREEN pays n euros back.

2nd prize: RED-WHITE pays 5 euros back.

3rd prize: GREEN-WHITE pays 2 euros back (i.e. the initial fee)

WHITE-WHITE gives nothing.

(a) Find the value of n if the game is fair. [4]

(b) Find the expected loss for the player after ten games if the first prize is $n = 25$ euros. [2]

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