HAEF IB - MATH SL
TEST - VECTORS
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SOLUTIONS

PART A [45 marks]

1. (a) $\boldsymbol{u} \bullet \boldsymbol{v} = 8 + 3 + p$ (A1)

For equating scalar product equal to zero (M1)

$8 + 3 + p = 0$

$p = -11$ A1 N3

(b) $|\boldsymbol{u}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$, 3.74 (M1)

$q\sqrt{14} = 14$ A1

$q = \sqrt{14}\ (=3.74)$ A1 N2

[6]

2. $p\boldsymbol{w} = p\boldsymbol{i} + 2p\boldsymbol{j} - 3pk$ (seen anywhere) (A1)

attempt to find $\boldsymbol{v} + p\boldsymbol{w}$ (M1)
eg $3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k} + p(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})$

collecting terms $(3 + p)\boldsymbol{i} + (4 + 2p)\boldsymbol{j} + (1 - 3p)\boldsymbol{k}$ A1

attempt to find the dot product (M1)
eg $1(3 + p) + 2(4 + 2p) - 3(1 - 3p)$

setting **their** dot product equal to 0 (M1)
eg $1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0$

simplifying A1
eg $3 + p + 8 + 4p - 3 + 9p = 0$, $14p + 8 = 0$

$P = -0.571\ \left(-\frac{8}{14}\right)$ A1 N3

[7]

3. (a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\overrightarrow{OR} = \begin{pmatrix} x \\ 3-3x \end{pmatrix}$ A1A1 N2

(b) $\overrightarrow{AB} \bullet \overrightarrow{OR} = x - 3(3 - 3x)$ A1

$\overrightarrow{AB} \bullet \overrightarrow{OR} = 0\ \ (10x - 9 = 0)$ M1

R is $\left(\frac{9}{10}, \frac{3}{10}\right)$ A1A1 N2

[6]

4. (a) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A1A1 N2

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A2A1A1 N4

[6]

5. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \left(= \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right)$ (M1)

$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ A2 N3

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ A1A1A1 N3

[6]

6. (a) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ (A1) (C1)

(b) $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{CD}$

$= \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1) (C1)

(c) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \overrightarrow{OD} - \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1)

$= \frac{1}{2}\overrightarrow{OD} + \frac{1}{2}\overrightarrow{OC}$ (A1) (C2)

Note: *Deduct [1 mark] (once only) if appropriate vector notation is omitted.*

[4]

7. evidence of equating vectors (M1)
eg $L_1 = L_2$

for any **two** correct equations A1A1
eg $2 + s = 3 - t$, $5 + 2s = -3 + 3t$, $3 + 3s = 8 - 4t$

attempting to solve the equations (M1)
finding **one** correct parameter ($s = -1$, $t = 2$) A1

the coordinates of T are (1, 3, 0) A1 N3

[6]

PART B [45 marks]

8. (a) (i) evidence of combining vectors (M1)

eg $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ (or $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$ in part (ii))

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ A1 N2

(ii) $\overrightarrow{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ A1 N1

(b) evidence of using perpendicularity ⇒ scalar product = 0 (M1)

e.g. $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$

$4 - 4(k - 5) + 4 = 0$ A1

$-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) A1

k = 7 AG N0

(c) $\overrightarrow{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ (A1)A1

evidence of correct approach *eg* $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ (M1)

$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ A1 N3

(d) **METHOD 1**

choosing appropriate vectors, $\overrightarrow{BA}, \overrightarrow{BC}$ (A1)

finding the scalar product M1

eg $-2(1) + 4(1) + 2(-1)$, $2(1) + (-4)(1) + (-2)(-1)$

$\cos A\hat{B}C = 0$ A1 N1

METHOD 2

$\overrightarrow{BC}$ parallel to $\overrightarrow{AD}$ (may show this on a diagram with points labelled) R1

$\overrightarrow{BC} \perp \overrightarrow{AB}$ (may show this on a diagram with points labelled) R1

$A\hat{B}C = 90°$

$\cos A\hat{B}C = 0$ A1 N1

[13]

9. (a) (i) Evidence of approach

eg $\vec{JQ} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$, $\vec{JQ} = \vec{JO} + \vec{OQ}$ M1

$\vec{JQ} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$ AG N0

(ii) $\vec{MK} = \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$ A1 N1

(b) (i) $\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$ or $\boldsymbol{r} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + t \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$ A2 N2

Note: *Award A1 if "**r** = " is missing.*

(ii) Evidence of choosing correct vectors $\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}, \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$ (A1)(A1)

Evidence of calculating magnitudes (A1)(A1)

eg $\sqrt{(-6)^2 + 7^2 + 10^2} = \sqrt{185}$ $\sqrt{6^2 + (-7)^2 + 10^2}$
$= \sqrt{185}$

$\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix} = -36 - 49 + 100$ (= 15) (accept – 15) (A1)

For evidence of substitution into the correct formula M1

eg $\cos \theta = \frac{15}{\sqrt{185}\sqrt{185}} \left(= \frac{15}{185} = 0.0811 \right)$

$\left(\text{accept} \frac{-15}{\sqrt{185}\sqrt{185}} \right)$

$\theta = 1.49$ (radians), 85.3° A1 N4

(c) **METHOD 1**

Geometric approach (M1)

Valid reasoning A2

eg diagonals bisect each other, $\vec{OD} = \vec{OM} + \frac{1}{2}\vec{MK}$

Calculation of mid point (A1)

eg $\left(\frac{6+0}{2}, \frac{0+7}{2}, \frac{0+10}{2}\right)$

$\vec{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix}$ (accept (3,3.5,5)) A1 N3

METHOD 2

Correct approach (M1)

eg $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \boldsymbol{t}\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + \boldsymbol{s}\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$

Two correct equations A1

eg $6 - 6t = 6s, 7t = 7 - 7s,\ 10t = 10s$

Attempt to solve (M1)

One correct parameter

$s = 0.5 \quad t = 0.5$ A1

$\vec{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix}$ (accept (3, 3.5, 5)) A1 N3

METHOD 3

Correct approach (M1)

eg $\begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + t\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$

Two correct equations A1

eg $-6t = 6s,\ 7 + 7t = 7 - 7s,\ 10 + 10t = 10s$

Attempt to solve (M1)

One correct parameter

$s = 0.5 \quad t = -0.5$ A1

$\vec{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix}$ (accept (3, 3.5, 5)) A1 N3

[16]

10. (a) speed = $\sqrt{3^2+4^2+10^2}$ (M1)

$= \sqrt{125} = 5\sqrt{5}$, 11.2, (metres per minute) A1 N2

(b) Let the velocity vector be $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Finding a velocity vector A2

eg $\begin{pmatrix} 3 \\ 16 \\ 39 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + 2\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $\begin{pmatrix} 3 \\ 16 \\ 39 \end{pmatrix} - \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix}$

Dividing by 2 to give $\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ A1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ AG N0

(c) (i) At Q, $\begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ (M1)

Setting up one correct equation A1

eg $3 + 3t = -5 + 4t$, $2 + 4t = 10 + 3t$, $7 + 10t = 23 + 8t$

$t = 8$ (A1)

Correct answer A1

eg after 8 minutes, 13:08 N3

(ii) Substituting for t (M1)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + 8\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$, or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + 8\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$

$x = 27$, $y = 34$, $z = 87$ or (27, 34, 87), or $\begin{pmatrix} 27 \\ 34 \\ 87 \end{pmatrix}$ A1 N2

(d) For choosing **both** direction vectors $\boldsymbol{d}_1 = \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$ and $\boldsymbol{d}_2 = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ (A1)

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = 104$, $|\mathrm{d}_1| = \sqrt{125}$, $|\mathrm{d}_2| = \sqrt{89}$ (A1)(A1)(A1)

$\cos\theta = \dfrac{104}{\sqrt{125}\sqrt{89}} = 0.98601...$ A1

$\theta = 0.167$ (radians) (accept $\theta = 9.59°$) A1 N3

[17]