HAEF IB - MATH SL
TEST - VECTORS
by Christos Nikolaidis

Marks:____/100

Grade: ______

Name:____________________________________

Date:________________

Questions

PART A [45 marks]

1. Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p.

(b) Given that $q|\mathbf{u}| = 14$, find the value of q. *[6 marks]*

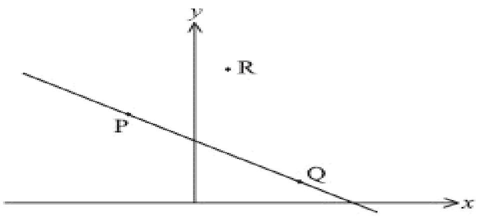
2. Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$. Find the value of p. *[7 marks]*

3. The line L passes through A (0, 3) and B (1, 0). The origin is at O. The point R $(x, 3 - 3x)$ is on L, and (OR) is perpendicular to L.

(a) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{OR}$.

(b) Use the scalar product to find the coordinates of R. *[6 marks]*

4. The points P(−2, 4), Q (3, 1) and R (1, 6) are shown in the diagram below.



(a) Find the vector $\overrightarrow{PQ}$.

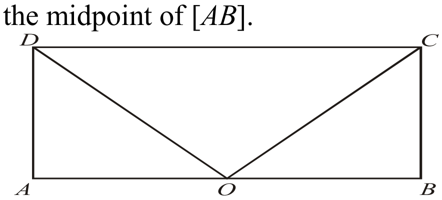
(b) Find a vector equation for the line through R parallel to the line (PQ). *[6 marks]*

5. The line L passes through the points A (3, 2, 1) and B (1, 5, 3).

(a) Find the vector $\overrightarrow{AB}$.

(b) Write down a vector equation of the line L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[6 marks]*

6. *ABCD* is a rectangle and *O* is the midpoint of [*AB*].



Express each of the following vectors in terms of $\overrightarrow{OC}$ and $\overrightarrow{OD}$

(a) *CD* (b) *OA* (c) *AD* (d) *AC* *[8 marks]*

7. The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T. *[6 marks]*

PART B [45 marks]

8. ***[maximum mark 13]***

Consider the points A (1, 5, 4), B (3, 1, 2) and D (3, k, 2), with (AD) perpendicular to (AB).

(a) Find (i) $\overrightarrow{AB}$; (ii) $\overrightarrow{AD}$, giving your answer in terms of k. *[3 marks]*

(b) Show that $k = 7$. *[3 marks]*

The point C is such that $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$.

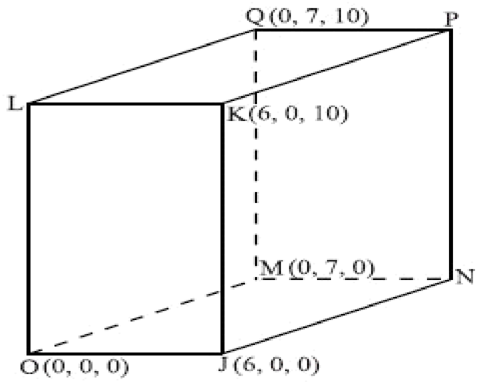
(c) Find the position vector of C. *[4 marks]*

(d) Find $\cos\ A\hat{B}C$. *[3 marks]*

9. ***[maximum mark 15]***

The diagram below shows a cuboid (rectangular solid) OJKLMNPQ.

The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).



(a) (i) Show that $\overrightarrow{JQ} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$. (ii) Find $\overrightarrow{MK}$. *[2 marks]*

(b) An equation for the line (MK) is $\boldsymbol{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$

(i) Write down an equation for the line (JQ) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Find the acute angle between (JQ) and (MK). *[8 marks]*

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D. *[5 marks]*

10. ***[maximum mark 17]***

Two model airplanes are each flying in a straight line (distance is in metres, time is in minutes).

At 13:00 the first model airplane is at the point (3, 2, 7). Its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$.

(a) Find the speed of the model airplane. *[2 marks]*

At 13:00 the second model airplane is at the point (– 5, 10, 23). After two minutes, it is at the point (3, 16, 39).

(b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ *[3 marks]*

(c) The airplanes meet at point Q.

(i) At what time do the airplanes meet? (ii) Find the position of Q. *[6 marks]*

(d) Find the angle θ between the paths of the two airplanes. *[6 marks]*