HAEF IB - MATH HL

TEST 9 – (PAPER 1: WITHOUT GDC)

VECTORS

by Christos Nikolaidis

Name:_________________________________

Date:________________

Marks:____/50

Grade: ______

Questions

1. *[maximum mark: 8]*

Let $\boldsymbol{u} = \begin{pmatrix} a \\ 2 \\ 3b+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -3 \\ 1 \\ b \end{pmatrix}$

Find the values of scalars *a* and *b* in each of the following cases

(a) $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$, and $a = 3b$ *[5 marks]*

(b) $\boldsymbol{u}$ is parallel to $\boldsymbol{v}$ *[3 marks]*

2. *[maximum mark: 10]*

Consider the parallelogram OABC. Let

$\boldsymbol{a}$ be the position vector of point A

$\boldsymbol{c}$ be the position vector of point C.

(a) Express $\overrightarrow{OB}$ and $\overrightarrow{CA}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$ *[2 marks]*

(b) Show that if $|\overrightarrow{OB}| = |\overrightarrow{CA}|$ then $\boldsymbol{a} \perp \boldsymbol{c}$ *[3 marks]*

(c) Show that if $\overrightarrow{OB} \perp \overrightarrow{CA}$ then $|\boldsymbol{a}| = |\boldsymbol{c}|$ *[3 marks]*

(d) If both the assumptions of (b) and (c) hold, describe geometrically the result. *[2 marks]*

3. *[maximum mark: 6]*

The Cartesian equations of the line *l* are

$$\frac{x-1}{-1}=\frac{y-3}{2}=\frac{z-6}{-1}$$

(a) Write down the vector equation of the line *l* *[1 mark]*

(b) Show that the point $A(4,-2,5)$ does not lie in the line *l*. *[1 mark]*

(c) Find the Cartesian equation of the plane containing line *l* and the point *A*. *[4 marks]*

4. *[maximum mark: 14]*

Consider the points A(1, 2, 1), B(0, –1, 2), C(1, 0, 2) and D(2, –1, –6).

(a) Calculate $\overrightarrow{AB}\times\overrightarrow{BC}$. *[3 marks]*

(b) Find the area of the triangle ABC. *[2 marks]*

(c) Find the Cartesian equation of the plane *Π* containing the points A, B and C. *[2 marks]*

(d) Find the distance of the point C to the line passing through A and B *[3 marks]*

(e) Find the distance from the point D to the plane *Π*. *[4 marks]*

5. *[maximum mark: 12]*

Consider the system of simultaneous equations.

$$\begin{aligned} x-2y-az&=b\\ 2x-y+3z&=2\\ 3x+y+2z&=-2 \end{aligned}$$

(a) Find the values of *a* and *b* for which the system has a unique solution. *[5 marks]*

(b) Find the values of *a* and *b* for which the system has no solution. *[2 marks]*

(c) Find the values of *a* and *b* for which the system has infinitely many solutions. Find the general solution. *[3marks]*

(d) Give a geometric description in cases (b) and (c). *[2 marks]*

HAEF IB - MATH HL

TEST 9 – (PAPER 2: WITH GDC)

VECTORS

by Christos Nikolaidis

Name:____________________________________

Date:________________

Marks:____**/50**

Grade: ______

Questions

1. *[maximum mark: 16]*

Consider the lines

$$L_1:\ r = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix} \qquad L_2:\ r = \begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}4\\7\\4\end{pmatrix}$$

and the planes

$$\Pi_1:\quad 2x + 3y + z = 7 \qquad \Pi_2:\quad 4x + 7y + 4z = 19$$

Find

(a) The angle between the lines L_1 and L_2 *[2 marks]*

(b) The angle between the planes Π_1 and Π_2 *[2 marks]*

(c) The angle between the line L_1 and the plane Π_2 *[2 marks]*

(d) The point of intersection of the lines L_1 and L_2 *[4 marks]*

(e) The point of intersection of the line L_1 and plane Π_1 *[3 marks]*

(f) The line of intersection of the planes Π_1 and Π_2 *[3 marks]*

2. *[maximum mark: 8]*

(a) Solve the following systems of simultaneous equations:

(i) $\begin{aligned}2x + 3y - z &= 2\\ x - y + 2z &= 1\\ 3x + 2y + 5z &= 5\end{aligned}$ (ii) $\begin{aligned}2x + 3y - z &= 2\\ x - y + 2z &= 5\\ 3x + 2y + z &= 6\end{aligned}$ (iii) $\begin{aligned}2x + 3y - z &= 2\\ x - y + 2z &= 5\\ 3x + 2y + z &= 7\end{aligned}$

[5 marks]

(b) Give a geometric interpretation of the solutions. *[3 marks]*

3. *[maximum mark: 7]*

Find the distance between the two points on the line L_1: $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ whose distance from point A(1,1,– 1) is $3\sqrt{5}$ units. *[7 marks]*

4. *[maximum mark: 10]*

Consider the lines

$$L_1: \ \frac{x-2}{-2} = 1 - y = 2z \qquad\qquad L_2: \ x = 2y, \ z = 3$$

(a) Show that the lines L_1 and L_2 intersect at the point A(-10,-5,3) *[2 marks]*

Let L_1' be the reflection of the line L_1 in the line L_2

(b) Find the angle between the line L_1 and L_1' *[4 marks]*

(c) Find the equation of the line L_1'. *[4 marks]*

5. *[maximum mark: 9]*

Consider the points A(1, 2, 1) and B(3, 0, 5).

(a) Find the Cartesian equation of the plane *Π* which consists of the equidistant points from A and B. *[4 marks]*

(b) Show that the points A and B lie in the plane Π_1: $x + 3y + z = 8$ and find the equation of the line *L* on the plane Π_1 which consists of the equidistant points from A and B. *[5 marks]*