HAEF IB - MATH HL

TEST 7 – (PAPER 1: WITHOUT GDC)

VECTORS

by Christos Nikolaidis

Name:_____

Date:_____

Marks:____/50

Grade: _____

Questions

1. *[maximum mark: 7]*

Let $\boldsymbol{u} = \begin{pmatrix} a \\ 2 \\ b+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -9 \\ 3 \\ b \end{pmatrix}$

Find the values of scalars a and b in each of the following cases

(a) $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ and $a = b$ *[4 marks]*

(b) $\boldsymbol{u}$ is parallel to $\boldsymbol{v}$ *[3 marks]*

2. *[maximum mark: 5]*

Given any two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that

$$|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 |\boldsymbol{b}|^2 - (\boldsymbol{a} \bullet \boldsymbol{b})^2.$$

3. *[maximum mark: 6]*

The vector equation of line l is given as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$.

Find the Cartesian equation of the plane containing line l and the point A(4,–2,5).

4. *[maximum mark: 12]*

Consider the system of simultaneous equations.

$$x - 2y - az = b$$
$$2x - y + 3z = 2$$
$$3x + y + 2z = -2$$

(a) Find the value of a for which the system has a unique solution. Find the solution in terms of a and b.

[4 marks]

(b) Find the value of a and the value of b for which the system does not have any solution.

[2 marks]

(c) Find the value of a and the value of b for which the system has infinitely many solutions. Find the general solution

[3marks]

(d) Give a geometric description for each case above.

[3 marks]

5. *[maximum mark: 20]*

Consider the points A(1, 2, 1), B(0, –1, 2), C(1, 0, 2) and D(2, –1, –6).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$. *[2 marks]*

(b) Calculate $\overrightarrow{AB} \times \overrightarrow{BC}$. *[2 marks]*

(c) Hence, or otherwise find the area of triangle ABC. *[2 marks]*

(d) Find the Cartesian equation of the plane P containing the points A, B and C. *[3 marks]*

(e) The line L through the point D is perpendicular to the plane P. Find the point of intersection E, of the line L and the plane P. *[5 marks]*

(f) Find the distance from the point D to the plane P. *[2 marks]*

(g) The point F is a reflection of D in the plane P. Find the coordinates of F. *[2 marks]*

(h) Find the volume of the pyramid ABCD *[2 marks]*

HAEF IB - MATH HL

TEST 7 – (PAPER 2: WITH GDC)

VECTORS

by Christos Nikolaidis

Name:_____

Date:_____

Marks:____/50

Grade: _____

Questions

1. *[maximum mark: 18]*

Consider the lines

$$L_1:\ r = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix} \qquad L_2:\ r = \begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}4\\7\\4\end{pmatrix}$$

and the planes

$$\Pi_1:\quad 2x + 3y + z = 7 \qquad \Pi_2:\quad 4x + 7y + 4z = 19$$

Find

(a) The angle between the lines L_1 and L_2 *[3 marks]*

(b) The angle between the planes Π_1 and Π_2 *[2 marks]*

(c) The angle between the line L_1 and the plane Π_2 *[2 marks]*

(d) The angle between the plane Π_1 and the y-axis *[2 marks]*

(e) The line of intersection of the planes Π_1 and Π_2 *[3 marks]*

(f) The point of intersection of the lines L_1 and L_2 *[3 marks]*

(g) The point of intersection of the line L_1 and plane Π_1 *[3 marks]*

2. *[maximum mark: 8]*

(a) Solve the following systems of simultaneous equations:

$$\begin{aligned} 2x + 3y - z &= 2\\ x - y + 2z &= 1\\ 3x + 2y + 5z &= 5 \end{aligned} \qquad \begin{aligned} 2x + 3y - z &= 5\\ x - y + 2z &= 7\\ 3x + 2y + z &= 5 \end{aligned} \qquad \begin{aligned} 2x + 3y - z &= 2\\ x - y + 2z &= 5\\ 3x + 2y + z &= 7 \end{aligned}$$

[5 marks]

(b) Describe geometrically the solutions.

[3 marks]

3. *[maximum mark: 4]*

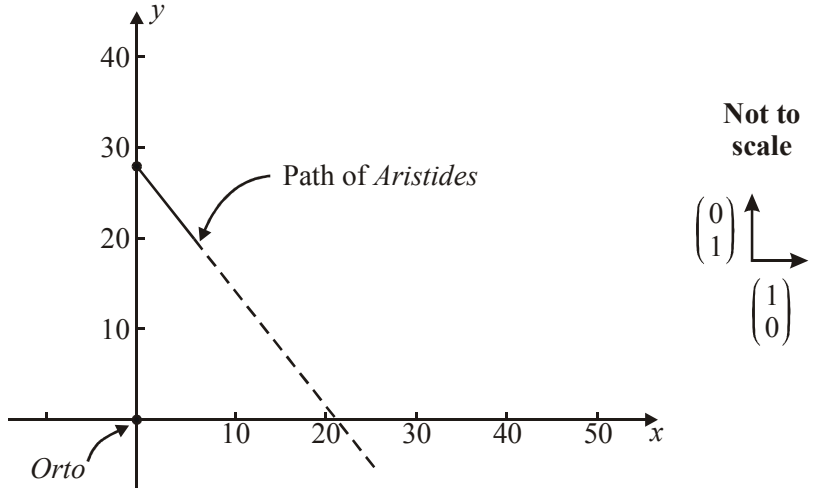
Find the angle between the lines

$$\frac{x-1}{2} = 1 - y = 2z \text{ and } x = y = 3z.$$

4. *[maximum mark: 20]*

In this question the vector $\begin{pmatrix}1\\0\end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix}0\\1\end{pmatrix}$ km represents a displacement due north.

The diagram shows the path of the oil-tanker *Aristides* relative to the port of *Orto*, which is situated at the point (0, 0).



The position of the *Aristides* is given by the vector equation $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\28\end{pmatrix}+t\begin{pmatrix}6\\-8\end{pmatrix}$

at a time t hours after 12:00.

(a) Find the position of the *Aristides* at 13:00. *[2 marks]*

(b) Find (i) the velocity vector; (ii) the speed of the *Aristides*. *[2 marks]*

(c) Find a Cartesian equation for the path of the *Aristides* in the form $ax + by = c$. *[3 marks]*

Another ship, the cargo-vessel *Boadicea,* is stationary, with position vector $\begin{pmatrix}18\\4\end{pmatrix}$ km.

(d) Show that the two ships will collide, and find the time of collision. *[3 marks]*

To avoid collision, the *Boadicea* starts to move at 13:00 with velocity vector $\begin{pmatrix}5\\12\end{pmatrix}$kmh^{-1}.

(e) Show that the position of the *Boadicea* for $t \geq 1$ is given by $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}13\\-8\end{pmatrix}+t\begin{pmatrix}5\\12\end{pmatrix}$ *[3 marks]*

(f) Find how far apart the two ships are at 15:00. *[3 marks]*

(g) The circular region around *Orto* with radius 20km is known as *Red zone*. Find the time for which the *Aristides* is within the *Red zone*. *[4 marks]*