INTERNATIONAL BACCALAUREATE

MATH HL

OPTION CALCULUS

EXERCISES

1. CONTINUITY AND DIFFERENTIABILITY

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A. Practice questions

1. Please visit the site
http://www.zweigmedia.com/RealWorld/calctopic1/canddex.html

There is an interactive exercise in continuity and differentiability

2. Determine the points of discontinuity and where the following functions are not differentiable.

$f(x) = |x+2|$

$g(x) = (x+1)^{2/3}$

$$h(x) = \begin{cases} x^2+1 & , \quad x<1 \\ 3 & , \quad x=1 \\ 2\sqrt{2x-1} & , \quad x>1 \end{cases}$$

$$k(x) = \begin{cases} x-1 & , \quad x \le 2 \\ 1 & , \quad 2<x\le 4 \\ x^2-8x+17 & , \quad x>4 \end{cases}$$

$$p(x) = \begin{cases} e^x - x & , \quad x \le 0 \\ \cos x & , \quad 0<x<\pi/2 \\ \sin 2x & , \quad x \ge \pi/2 \end{cases}$$

$$q(x) = \begin{cases} -2x-2 & , \quad x<-1 \\ x^2-1 & , \quad -1\le x\le 1 \\ 2x-2 & , \quad x>1 \end{cases}$$

B. Past paper questions

3. The function $f : \mathbb{R} \to \mathbb{R}$ is defined as $f : x \to \begin{cases} 1 & , x<0 \\ 1-x, & x \ge 0 \end{cases}$.

By considering limits, prove that f is

(a) continuous at $x = 0$; ***[2]***

(b) not differentiable at $x = 0$. ***[3]***

4. Let $f(x) = 2x + |x|$, $x \in \mathbb{R}$.

(a) Prove that f is continuous but not differentiable at the point $(0, 0)$. ***[7]***

(b) Determine the value of $\int_{-a}^{a} f(x)\,dx$ where $a > 0$. ***[3]***

5. The function f is defined by $f(x) = \begin{cases} e^{-x^2}\left(-x^3 + 2x^2 + x\right), & x \le 1 \\ ax + b, & x > 1 \end{cases}$, where a and b are constants.

(a) Find the exact values of a and b if f is continuous and differentiable at $x = 1$. ***[8]***

6. The function f is defined by

$$f(x) = \begin{cases} 2x - 1, & x \le 2 \\ ax^2 + bx - 5, & 2 < x < 3 \end{cases}$$

where $a, b \in \mathbb{R}$.

(a) Given that f and its derivative, f', are continuous for all values in the domain of f, find the values of a and b. ***[6]***

(b) Show that f is a one-to-one function. ***[3]***

(c) Obtain expressions for the inverse function f^{-1} and state their domains. ***[5]***

7. The weight in kilograms of a dog, t weeks after being bought from a pet shop, can be modelled by the following function:

$$w(t) = \begin{cases} 2 + ct & 0 \le t \le 5 \\ 16 - \dfrac{35}{t} & t > 5 \end{cases}$$

(b) Given that $w(t)$ is continuous, find the value of c. ***[2]***

(c) Write down

(i) the weight of the dog when bought from the pet shop;

(ii) an upper bound for the weight of the dog. ***[2]***

(d) Prove from first principles that $w(t)$ is differentiable at $t = 5$. ***[6]***